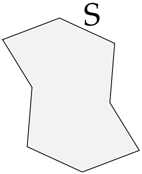
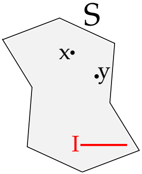
Theorem (Masur's criterion, '92)

Suppose that the vertical geodesic flow of a translation surface S is minimal (orbits are dense) but not uniquely ergodic (orbits are not equidistributed). Then the orbit $g_t \cdot S$ eventually leaves any compact set of the stratum ("diverges to infinity").

Proof of Masur's criterion

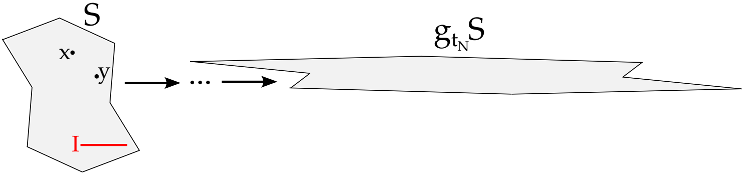


Proof of Masur's criterion



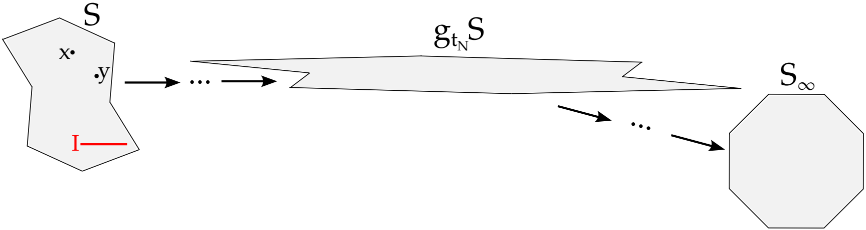
- let $x, y \in S$ be s.t. their vertical orbits visit some horizontal interval $I \subset S$ with different asymptotic frequencies;

Proof of Masur's criterion



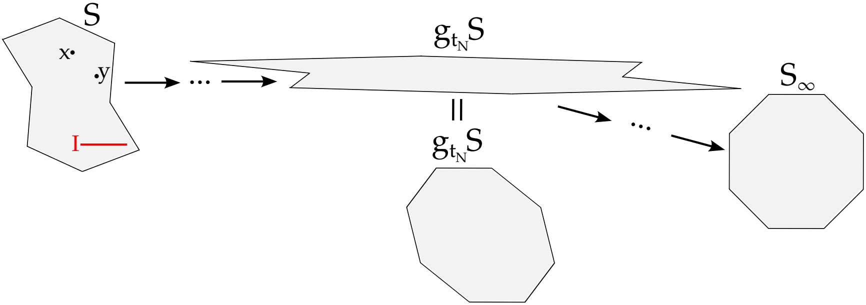
- let $x, y \in S$ be s.t. their vertical orbits visit some horizontal interval $I \subset S$ with different asymptotic frequencies;
- by contradiction, assume that $\exists t_N \to \infty$ s.t. $g_{t_N} \cdot S \to S_\infty$;

Proof of Masur's criterion



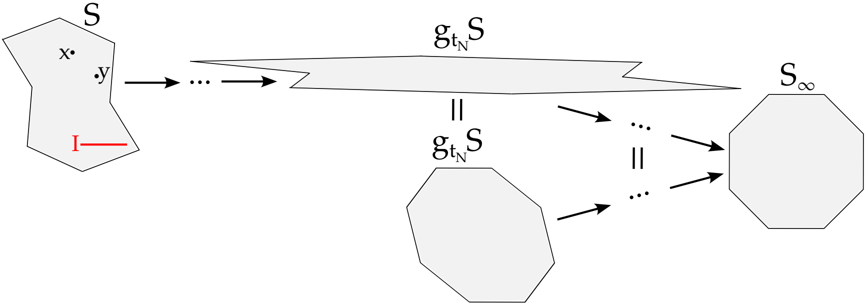
- let $x, y \in S$ be s.t. their vertical orbits visit some horizontal interval $I \subset S$ with different asymptotic frequencies;
- by contradiction, assume that $\exists t_N \to \infty$ s.t. $g_{t_N} \cdot S \to S_\infty$;

Proof of Masur's criterion



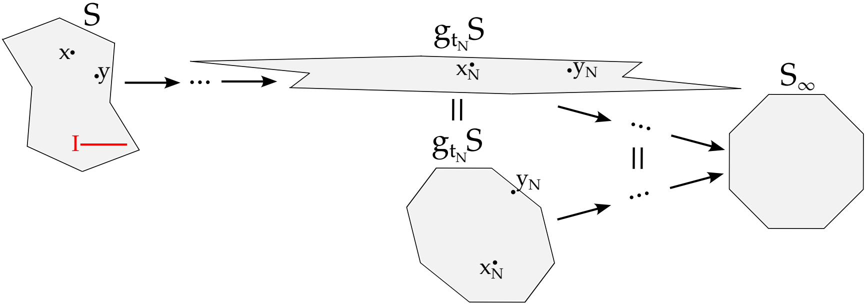
- let $x, y \in S$ be s.t. their vertical orbits visit some horizontal interval $I \subset S$ with different asymptotic frequencies;
- by contradiction, assume that $\exists t_N \to \infty$ s.t. $g_{t_N} \cdot S \to S_\infty$;

Proof of Masur's criterion



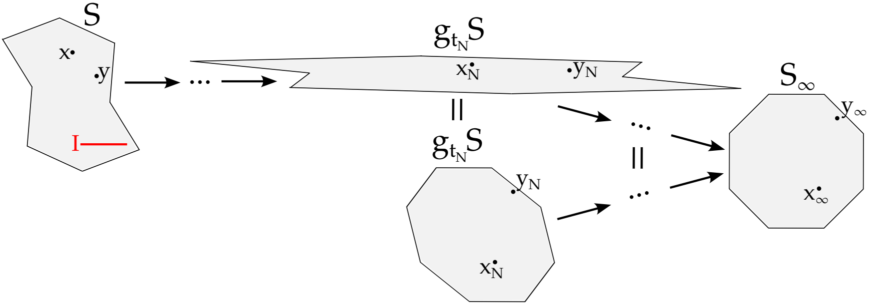
- let $x, y \in S$ be s.t. their vertical orbits visit some horizontal interval $I \subset S$ with different asymptotic frequencies;
- by contradiction, assume that $\exists t_N \to \infty$ s.t. $g_{t_N} \cdot S \to S_\infty$;

Proof of Masur's criterion



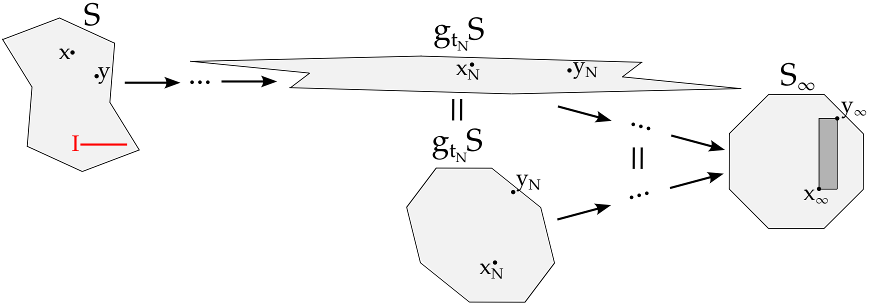
- let $x, y \in S$ be s.t. their vertical orbits visit some horizontal interval $I \subset S$ with different asymptotic frequencies;
- by contradiction, assume that $\exists t_N \to \infty$ s.t. $g_{t_N} \cdot S \to S_\infty$;

Proof of Masur's criterion



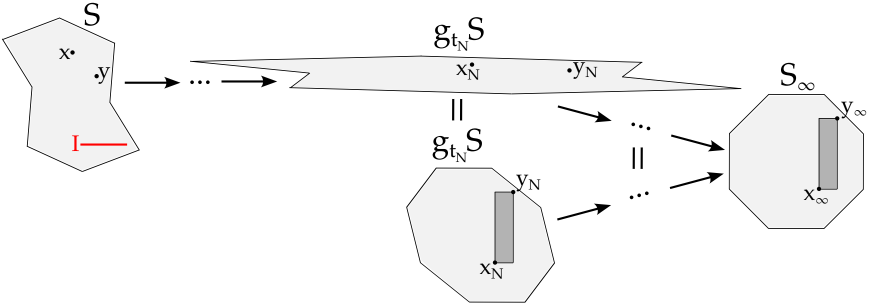
- let $x, y \in S$ be s.t. their vertical orbits visit some horizontal interval $I \subset S$ with different asymptotic frequencies;
- by contradiction, assume that $\exists t_N \to \infty$ s.t. $g_{t_N} \cdot S \to S_\infty$;
- we may suppose that $x_N = g_{t_N} x$ and $y_N = g_{t_N} y$ converge to some $x_\infty, y_\infty \in S_\infty$;

Proof of Masur's criterion



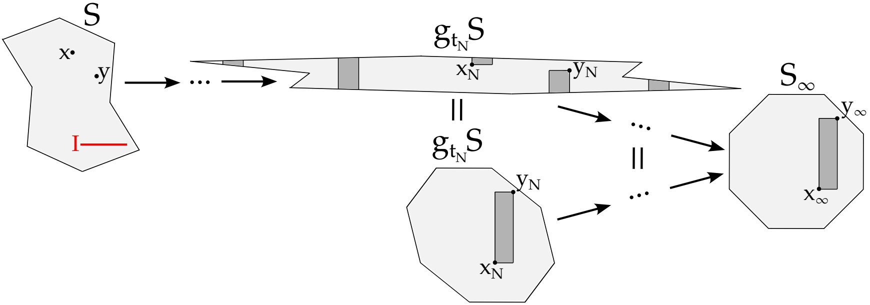
- let $x, y \in S$ be s.t. their vertical orbits visit some horizontal interval $I \subset S$ with different asymptotic frequencies;
- by contradiction, assume that $\exists t_N \to \infty$ s.t. $g_{t_N} \cdot S \to S_\infty$;
- we may suppose that $x_N = g_{t_N} x$ and $y_N = g_{t_N} y$ converge to some $x_\infty, y_\infty \in S_\infty$;

Proof of Masur's criterion



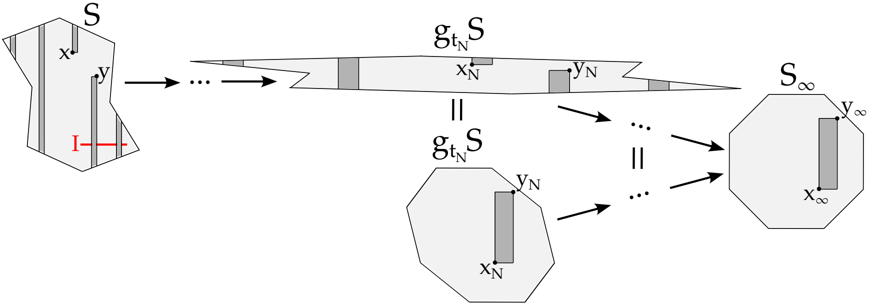
- let $x, y \in S$ be s.t. their vertical orbits visit some horizontal interval $I \subset S$ with different asymptotic frequencies;
- by contradiction, assume that $\exists t_N \to \infty$ s.t. $g_{t_N} \cdot S \to S_\infty$;
- we may suppose that $x_N = g_{t_N} x$ and $y_N = g_{t_N} y$ converge to some $x_\infty, y_\infty \in S_\infty$;

Proof of Masur's criterion



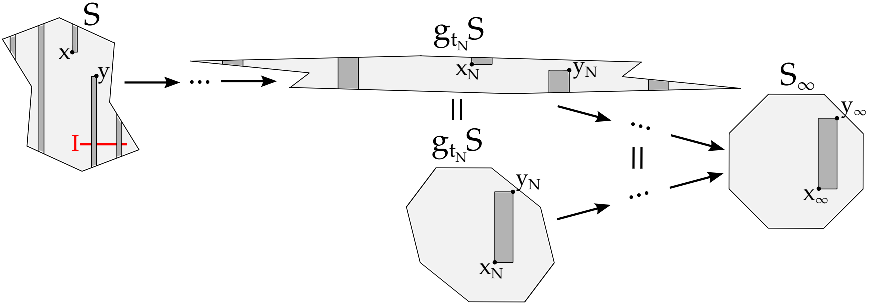
- let $x, y \in S$ be s.t. their vertical orbits visit some horizontal interval $I \subset S$ with different asymptotic frequencies;
- by contradiction, assume that $\exists t_N \to \infty$ s.t. $g_{t_N} \cdot S \to S_\infty$;
- we may suppose that $x_N = g_{t_N} x$ and $y_N = g_{t_N} y$ converge to some $x_\infty, y_\infty \in S_\infty$;

Proof of Masur's criterion



- let $x, y \in S$ be s.t. their vertical orbits visit some horizontal interval $I \subset S$ with different asymptotic frequencies;
- by contradiction, assume that $\exists t_N \to \infty$ s.t. $g_{t_N} \cdot S \to S_\infty$;
- we may suppose that $x_N = g_{t_N} x$ and $y_N = g_{t_N} y$ converge to some $x_\infty, y_\infty \in S_\infty$;
- vertical orbits of x and y are sides of a very tall and very thin rectangle in $S \Rightarrow$ they intersect I equal number of times (± 1);

Proof of Masur's criterion



- let $x, y \in S$ be s.t. their vertical orbits visit some horizontal interval $I \subset S$ with different asymptotic frequencies;
- by contradiction, assume that $\exists t_N \to \infty$ s.t. $g_{t_N} \cdot S \to S_\infty$;
- we may suppose that $x_N = g_{t_N} x$ and $y_N = g_{t_N} y$ converge to some $x_\infty, y_\infty \in S_\infty$;
- vertical orbits of x and y are sides of a very tall and very thin rectangle in $S \Rightarrow$ they intersect I equal number of times (± 1);
- $N \to \infty$ gives contradiction. □